FINAL REPORT ON

"CONSTITUTIVE EQUATIONS IN TWO-DIMENSIONAL FLOW"

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Submitted by

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"CONSTITUTIVE EQUATIONS IN TWO-DIMENSIONAL FLOW"

CONTRACT NUMBER: NsG 671

PERIOD OF CONTRACT: September 1964 - August 1969

CONTRACTOR: Department of Chemical and Metallurgical Engineering
University of Tennessee, Knoxville, Tennessee

INVESTIGATORS:

Principal:

D.C.Bogue, Professor of Chemical Engineering

Associates:

	approx. support period:
(1) E. B. Adams	3 years
(2) R. L. Boles	2 years
(3) H. L. LaNieve	1 year
(4) J. O. Casey	1 1/2 years
(5) I. J. Chen	1 1/2 years

OBJECTIVES:

To develop and test constitutive equations for viscoelastic fluids, especially in two-dimensional flows.

SUMMARY OF RESEARCH ACCOMPLISHED

In a general way the research has been concerned with developing constitutive (stress-strain) equations for viscoelastic materials, with particular emphasis on accelerative ("two-dimensional") flows. Such flows are of very general, practical significance in that all extrusion processes involve the squeezing down of materials from large cross-sections to small ones. The research is summarized below in the order in which the projects developed; the overall objectives and results are noted very briefly, with reference being made to the published articles for the details. Actually, due to the step-funding arrangements, Mr. Chen and Mr. Casey are presently being supported, in part, from the contract and will continue to be through August 1969. In terms of the experimental work and overall scope, however, the research has been substantially completed and will be reported at this time. The finishing details will be available with the publication of their work.

1. Birefringent Studies in Entrance Flows; Analysis of Entrance Flows (Adams, Fields *)

In work started about seven years ago, we have measured stress-birefringent patterns in viscoelastic flows, at sharp-edged and tapered channel ("two-dimensional") entrances. Polarized light is passed

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* Mr. T.R. Fields' work contributed to the project although he was supported by a NASA Traineeship and not by the contract.

through the flowing test material to produce optical patterns which, with certain assumptions, can be quantitatively converted to stresses. The present work has centered on interpreting these stresses in terms of viscoelastic theory (Adams, Whitehead and Bogue). As an outgrowth of this work, more general theoretical analysis of converging-diverging flow was carried out using full integral viscoelastic theory (Adams and Bogue). The most recent work has been concerned with birefringent measurements of much more elastic fluids at entrances. A very striking stress pattern, resembling a shock wave, has been observed and is not fully understood at the present time (Fields and Bogue).

2. Pressure Drops in Entrance Flows (LaNieve, Boles)

In two projects, we dealt directly with an important although very complicated practical problem: that of predicting pressure drops in tube entrances. We obtained some help from the theoretical analyses noted above in establishing the parameters and correlating groups, but the final correlations are best described as empirical. In the first work we correlated pressure drops at sharp-edged entrances with an independent rheological measurement, normal stresses (LaNieve and Bogue). In later work we also dealt with tapered entrances using a similar correlation scheme (Boles and Bogue).

3. Rheological Properties of Polymer Melts (Chen)

Basic studies of the rheological properties of polymer melts

(polyethylenes, polystyrene, polypropylene) have been carried out on the Weissenberg Rheogoniometer. The emphasis has been on strongly non-linear elastic effects, by starting and stopping the applied strain rate as a function of time. These data have been analyzed with integral-type viscoelastic theories. The objective of this work has been to find distinctive rheological differences between polymer melts and to use the parameters from such measurements in correlating complex flow effects, such as melt fracture.

4. Macroscopic Energy Balances and Stability Analyses (Casey)

In contrast to the detailed differential analysis of entrance flows carried out by Adams, we have also set down and solved overall, macroscopic balances. The hope was to be able to associate the large entrance pressure drops with the elastic energy stored by the fluid. The elastic energy can be calculated, given certain constitutive theory, before and after the entrance without knowledge of the details of the flow at the entrance itself. The conclusion was that this approach is not generally satisfactory: that is, that the large entrance pressure drops are due to irreversible losses and not to stored elastic energy. It was possible, however, to state the overall balances in a systematic manner such that they may be used in other problems. Casey's work has now turned to a rather different area: that of finding well-defined rheological experiments (such as extensional flow) which can be associated directly with entrance flows, as a route to characterizing melt fracture.

PAPERS RESULTING FROM THE CONTRACT

Papers published or accepted for publication or presentation:

1. Adams, E.B., J.C. Whitehead and D.C. Bogue, "Stresses in a Viscoelastic Fluid in Converging and Diverging Flow", AIChE Journal, 11, 1026-32 (1965). (partly supported by the contract)

2. LaNieve, H.L. and D.C. Bogue, "Correlation of Capillary Entrance Pressure Drops with Normal Stresses", J. Applied Polymer Science, 12, 353 (1968). (partly)

3. Fields, T.R., Jr., and D.C. Bogue, "Stress Birefringent Patterns at a Sharp-Edged Entrance", Trans. Soc. Rheology, 12:1, 39-55 (1969). (partly)

4. Adams, E.B., and D.C. Bogue, "Viscoelasticity of Accelerative Flows: A Simplified Integral Theory", AIChE Journal, in press. (fully)

5. Boles, R.L., and D.C. Bogue, "Entrance Flows of Polymeric Materials, Part I. Pressure Drop", to be presented and preprinted for the Society of Plastics Engineers Meeting, May 1969; submitted to Polymer Engineering and Science. (partly)

Papers proposed:

1. Chen, I.J. and D.C. Bogue, "Rheological Properties of Polymer Melts; and Development of Constitutive Equations for Polymer Melts". (partly)

2. Casey, J.O. and D.C. Bogue, "Overall Energy Balances in Viscoelastic Fluids and Analysis of the Tube Entrance Problem". (fully)

THESES COMPLETED OR IN PROGRESS IN CONNECTION WITH THE ABOVE WORK

1. Adams, E.B., "A Simplified Constitutive Equation for Two-Dimensional Viscoelastic Flow", PhD thesis, August 1967.

2. LaNieve, H.L., "Correlation of Capillary Entrance Pressure Drops with Normal Stress Data", PhD thesis, March 1966.

3. Fields, T.R., Jr., "Stress-Birefringent Patterns at a Sharp-Edged Entrance", MS thesis, March 1967.

4. Boles, R.L., "An Experimental Study of Viscoelastic Flow in a Conical Entrance Region", PhD thesis, December 1967.

5. Casey, J.O., PhD thesis, in progress.

6. Chen, I.J., PhD thesis, in progress.